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The Routledge International Handbook of Dyscalculia and Mathematical Learning Difficulties

Edited by Steve Chinn

The Routledge International Handbook of Dyscalculia and Mathematical Learning Difficulties

Mathematics plays an important part in every person's life, so why isn't everyone good at it? *The Routledge International Handbook of Dyscalculia and Mathematical Learning Difficulties* brings together commissioned pieces by a range of hand-picked influential, international authors from a variety of disciplines, all of whom share a high public profile. More than fifty experts write about mathematics learning difficulties and disabilities from a range of perspectives and answer questions such as:

- What are mathematics learning difficulties and disabilities?
- What are the key skills and concepts for learning mathematics?
- How will IT help, now and in the future?
- What is the role of language and vocabulary?
- How should we teach mathematics?

By posing notoriously difficult questions such as these and studying the answers *The Routledge International Handbook of Dyscalculia and Mathematical Learning Difficulties* is the authoritative volume, and essential reading, for academics in the field of mathematics. It is an incredibly important contribution to the study of dyscalculia and mathematical difficulties in children and young adults.

Steve Chinn is an independent consultant, researcher and writer who presents papers and contributes to conferences world-wide. He has delivered training courses for teachers, psychologists, parents and support assistants in over thirty countries. Among his other books are the award-winning *The Trouble with Maths* (now in its second edition, 2011) and *More Trouble with Maths* (2012), both published by Routledge.

The Routledge International Handbook of Dyscalculia and Mathematical Learning Difficulties

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Arithmetic difficulties of children with hearing impairment

Gowramma I. P.

Introduction

It has been well documented that hearing impairment in early childhood has an adverse effect on speech and language development. Impairment in the development of speech, language and oral communication skills is known to hinder the educational development of children. Paul and Jackson (1993) reported that differences in language abilities consequent to hearing impairment affects a student's ability to perform in traditional academic areas. The delay in educational achievement of children with hearing impairment (CWHI) compared to their hearing peers has been noted to occur since the educational system is highly language based. Paul and Quigley (1994) reported that students with complete or partial hearing impairment have considerable difficulty succeeding in an educational system that depends primarily on spoken word and written language to transmit knowledge. According to Flexer (1999), irrespective of the degree of impairment, hearing loss if unmanaged can have a negative impact on the development of academic competencies. A decade before this, Greenberg and Kusche (1989) and Martin (1985) had observed that children with mild-to-moderate hearing loss achieve below expectations, based on their performance on a test of cognitive ability.

Literature in the area of mathematics learning of CWHI is rich. A review of literature between 1980 and 2013 indicates that CWHI lag behind their hearing peers in mathematics achievement (Swanwick, Oddy, & Roper, 2005). Disparity in the performance of CWHI and their hearing counterparts was noticed in mathematics achievement on the Stanford Achievement Test. Compared to their reading performance they performed at a higher grade level in mathematics though the performance was below grade level (Stewart & Kluwin, 2001). Paranjape (1998) observed similar results when she compared the performance of CWHI with normally hearing children (NHC) based on achievement tests which showed difference

in language performance but not in mathematics. Gowramma (2006) found no significant difference in the performance of CWHI when compared with a matched group of NHC based on the scores of an arithmetic diagnostic test. Wood et al. (1983), based on the performance of standardized tests, concluded that hearing loss is not the direct cause of difficulties in mathematics as 15% of the participants who had profound hearing impairment performed at average and above average levels. Better performance of mainstreamed CWHI is recorded (Kluwin & Moores, 1985; Wood, Wood, Kinsmill, French, & Howarth, 1984) compared to those segregated in special schools. Differences in expectations, exposure, trained teachers, parental involvement and support services were seen as factors bringing in the difference. Meadow (1980) noticed that the learning process is slower among CWHI, though their mathematical reasoning is on par with normal hearing children. Hyde, Zevenbergen and Power (2003), who studied the performance of students with hearing impairment on arithmetic word problems, found them to perform poorly.

Researchers have explored reasons for poor performance of CWHI in mathematics. Limited incidental learning and reinforcement (Gregory, 1998), delay in early access to mathematical conversation (Pau, 1995), limited auditory experience affecting short-term memory (Epstein, Hillegeist, & Grafman, 1994), lack of spoken language which develops inner speech (Hitch, Arnold, & Phillips, 1983) are examples. Stewart and Kluwin (2001) opined that the reasons why mathematics was challenging for CWHI were varied and complex. The reasons ranged from the inability to learn from experiences outside the classroom to utilization of cognitive abilities in the classroom such as the inability to assign meaning and language to mathematical problems. Earlier, Barton (1995) found that the cognitive concepts which involved specific language related to volume, shape, size, comparisons, measurement and reasoning were particularly difficult for CWHI to grasp, which may affect mathematics performance. The above reasons are noted to pose hurdles in information processing and mastering fundamental operations.

Strengths of CWHI in learning mathematical concepts and skills have been researched. It was found that the oral CWHI performed on par with hearing children in number reproduction on the temporal tasks and outperformed in spatial tasks (Zarfaty, Nunes, & Bryant, 2004). Thus, they concluded that the difficulties of CWHI were not consequent to a delay in number representation. A study which compared the performance of CWHI with NHC by Yathiraj et al. (2013) observed that the performance of the two groups did not differ when the tasks were presented visually in a pre-arithmetic school readiness test. Nunes and Moreno (1997) highlight visualization of counting as a factor for learning arithmetic. Vijayalakshmi and Gowramma (2010) confirmed significant improvement in the performance of CWHI when they were taught mathematics through a play-way method which relied on visual and concrete experience.

Nunes and Moreno (1998) reviewed data-based literature and concluded that it does not support a causal link between hearing loss and the difficulties experienced in developing mathematical knowledge and skills. Authors have suggested that hearing impairment should be considered more of a risk factor in learning numerical concepts. Mousley and Kelly (1998) stressed the importance of meta-cognitive skills,

repetitive practice, active participation, interactive discussion and evaluative feedback in learning mathematics for CWHI.

Quantitative skills are considered as an important aspect of language development. In addition to basic language concept, vocabulary specific to mathematics is essential in order to learn formal mathematics in school. CWHI, who are usually deficient in these areas, would encounter certain difficulties in learning mathematics. The findings from the literature bring to light that though there are some instances where CWHI exhibited better performance, generally they are found to be performing below their age-appropriate grade levels. Some important conclusions can be drawn based on the review of research. First of all, not all CWHI are weak in mathematics performance. Next, hearing impairment does pose a risk for learning mathematics. Finally, fewer opportunities to learn from the socio-economic context impose a hurdle to learning school mathematics. A review of the literature indicates an encouraging point of view which maintains that CWHI can achieve their potential if the environment, instruction, and materials are appropriate. We need to be able to identify more precisely the areas of specific difficulties and errors in arithmetic of CWHI to plan effective ways to help them acquire the important skills. This study is thus proposed to analyse the difficulties experienced and errors committed in arithmetic by CWHI studying in special schools and compare them with a matched group of NHC.

Participants

Participants were selected from two special schools (coded as SS, N = 47) meant for CWHI. The special schools are residential, run by nongovernmental organizations. This facility is usually availed by children with disabilities from economically weaker sections residing in rural areas where parents either are uneducated or with minimum educational background. The range of hearing loss of the participants was from mild to profound. Only those children who were reported to have no additional disability were selected. To compare their performance NHC from a regular school (coded as RS, N = 18) matching the special schools with respect to economic background and the literacy level of parents were chosen. It was decided to select grades IV and V considering that all the children were exposed to school instruction for at least three years. It was ensured that none of the students had any obvious disability as reported by their teachers. All the schools were located in Mysore city in the State of Karnataka where Kannada, the regional language, is the medium of instruction.

Description of the tool

Arithmetic Diagnostic Test (ADT) for Primary School Children (Ramaa, 1994) diagnoses the specific difficulties encountered by children in primary schools in grade I through IV. The test covers three major areas of arithmetic: namely, number concept, arithmetic operations and reasoning. Since it is a diagnostic test, it includes problems that represent each type and subtype of tasks that fall under each of the major areas. Thus the test is quite comprehensive in identifying the strengths and weaknesses of the individual child.

Due weighting is given to different types of tasks. Each subtype of a task is represented by two items in the case of arithmetic processes and reasoning. This helps in thorough diagnosis of the difficulties faced by the children in dealing with particular subtypes of an arithmetic task. The sub-items and the items are arranged in the order of increasing level of difficulty within the different section of items as well as between the sections.

Selection of items and administration of the test

Some of the items from the test were eliminated based on the opinions of teachers in the participant schools, as they were not covered under the school syllabus. Testing was carried out in small groups within the school premises, using areas that were free from visual and auditory distracters. The rooms selected were located away from sources of noise and had adequate natural lighting. As the test was given at the beginning of the academic year, students studying in grade IV were considered to be performing at grade III level and those studying in grade V were considered to be performing at grade IV level.

Results

Comparison of scores between CWHI and NHC

An independent t-test was performed to check if the difference in scores was statistically significant. The t-test indicated that there was no significant difference between the CWHI and the NHC [$F(63) = 1.64, p > 0.05$] for the overall scores. Since the sample size of the two groups was unequal, the result of independent t-test was cross checked with the non-parametric Mann-Whitney U test. Similar results were obtained through both the statistical procedures ($z = 1.36, p > 0.05$).

Even though 'z' is not significant, descriptively differences are observed. Table 8.1 depicts the descriptive statistic of the total test scores for the two groups. It can be seen that the mean score of the NHC is higher than that obtained by the CWHI. The standard deviation is high for both the groups suggesting diversity. It is seen that the deviation is greater among CWHI.

Analysis of the specific difficulties experienced in both the groups

For the purpose of analysing the difficulties for each of the criterion measures selected from the diagnostic tool, a score of one was given to a correct item and zero to a wrong or not-attempted item. On the basis of the raw score obtained for each

Table 8.1 Mean and SD of the total test scores of the two groups

Groups	N	*Mean	SD
CWHI	47	93.00	40.45
NHC	18	107.44	35.37

Note: * Maximum possible score = 202

Table 8.2 Number concept – 1

Criterion measures →	Counting		Writing the given numbers in words – up to 2 digits		Writing the given numbers in words – more than 2 digits		Writing the numbers from the words given – up to 2 digits		Writing the numbers from the words given – more than 2 digits						
	M	PA	NA	M	PA	NA	M	PA	NA	M	PA	NA			
SS	100	-	-	100	-	-	40	49	11	72	28	-	4	43	53
RS	100	-	-	100	-	-	56	39	6	89	11	-	39	50	11

School

Table 8.3 Number concept – 2

School	Criterion measures →		Sequential representation of numbers in blanks given – less than 2 digits			Sequential representation of numbers in blanks given – more than two digits			Numbers less than and greater than a given number			Arranging the numbers in ascending order – less than 2 digits			Arranging the numbers in ascending order – more than 2 digits			
	M	PA	NA	M	PA	NA	M	PA	NA	M	PA	NA	M	PA	NA	M	PA	NA
SS	96	4	-	96	4	-	19	47	34	74	74	9	17	43	15	43	15	43
RS	89	-	11	72	6	22	83	-	17	67	17	17	17	44	22	44	22	33

Table 8.4 Addition

Criterion measures →	Addition of single-digit numbers			Addition of 2-digit numbers without carry over			Addition of 2-digit numbers with carry over			Addition of numbers more than 2 digits			Problem solving involving addition based on verbal numerical information			Problem solving involving addition based on verbal numerical information – when more information is given		
	M	PA	NA	M	PA	NA	M	PA	NA	M	PA	NA	M	PA	NA	M	PA	NA
SS	100	-	-	81	15	4	72	9	19	47	42	11	42	26	32	-	-	100
RS	100	-	-	83	17	-	44	22	33	39	61	-	33	39	28	-	-	100

School

Table 8.5 Subtraction

Criterion measures →	Subtracting single digits		Subtracting 2-digit numbers without borrowing		Subtracting 2-digit numbers with borrowing		Subtracting more than 2-digit numbers		Problem solving involving subtraction based on verbal numerical information – numerical information involving more than one operation		Problem solving involving subtraction based on verbal numerical information – numerical information involving more than one operation						
	M	PA	M	NA	M	PA	M	NA	M	PA	M	NA	PA	NA			
SS	72	13	68	11	21	45	6	49	13	45	42	15	25	60	–	–	100
RS	83	6	61	28	11	33	28	39	22	56	22	22	39	39	–	–	100

Table 8.6 Multiplication

School	Multiplication of 1-digit numbers			Multiplication of 2-digit numbers by 1-digit numbers			Multiplication of more than 2-digit numbers			Problem solving involving multiplication based on verbal numerical information		
	M	PA	NA	M	PA	NA	M	PA	NA	M	PA	NA
SS	60	30	11	23	40	47	4	23	72	-	15	85
RS	89	-	11	44	39	17	17	44	39	-	50	50

Table 8.7 Division

Criterion measures → School	Division of less than 2-digit numbers in one step			Division of more than 2-digit numbers with more than one step			Problem solving involving division based on verbal numerical information		
	M	PA	NA	M	PA	NA	M	PA	NA
SS	26	26	49	4	26	70	–	–	100
RS	28	56	11	11	50	33	–	11	89

of the criterion measures, children who scored 75% and above were classified as masters (M), below 75% as partial achievers (PA) and those who scored '0' were grouped under non-achievers (NA). The number of masters, partial achievers and non-achievers were noted for both the groups. The total number of children in each group was converted into a percentage for the purpose of comparison.

Percentage of children who are masters (M), partial achievers (PA) and non-achievers (NA) in the criterion measures of number concept, addition, subtraction, multiplication and division from special schools (SS) and regular school (RS) are given in the Tables 8.2 to 8.7.

Major findings and discussion at the stage of analysis of difficulties

Number concept

As observed in Table 8.2, for the first two criterion measures only, 100% mastery is achieved by both the groups. For the next three criterion measures in Table 8.2, there are more masters and fewer non-achievers among the NHC. However mastery is achieved by a significant number of CWHI also, except for writing the numbers from words for numbers that have more than two digits. It is interesting to note that among CWHI the number of masters is higher for writing the given number in words compared to writing the numbers in digits from the words given. For the latter task more verbal information in sequence has to be decoded compared to the former. Craig and Gordon (1988) observed that verbal sequential skills in cognitive functioning were below average among CWHI. The difficulty might also be due to limited auditory experience effecting short-term memory (Epstein et al., 1994) and lack of spoken language which leads to inner speech (Hitch et al., 1983).

Table 8.3 shows that CWHI have performed better than the NHC in the sequential representation of numbers where there are more masters and no non-achievers. But among the NHC there are still non-achievers. As this relies on visuo-spatial skills CWHI had an advantage over their hearing counterparts (Zarfaty et al., 2004; Craig & Gordon, 1988). In contrast, recalling numbers greater and lesser than the number given was accomplished better by NHC. Cognitive concepts involving language related to volume, size and comparisons are particularly difficult for CWHI (Barton, 1995). This, being a quantitative concept relating to temporal perception requiring the pupil to recall the number coming after and before a given number, meant that CWHI faced difficulty. In addition, the linguistic demand on processing 'lesser than' and 'greater than' might be demanding for CWHI. To arrange the numbers in ascending order, knowledge of sequence and comparison in terms of quantity are both essential. A more or less similar level of difficulty is faced by both the groups for the task.

Fundamental operations

Little difference is observed in the level of difficulty by both the groups for all the criterion measures of addition and subtraction. CWHI might be performing the learnt operation mechanically where they are well practiced. In multiplication and division

there are more masters and fewer non-achievers among NHC. This indicates that the difficulty of CWHI increases as the higher order thinking is involved in fundamental operation. The mathematical reasoning of CWHI is on par with hearing children but the learning process is slow (Meadow, 1980).

Arithmetic reasoning

Surprisingly mastery was attained by a slightly higher percentage of CWHI compared to NHC for the statement problems in addition. The difficulty level is almost the same for both the groups with the other operations, especially when the descriptive demand increased. This phenomenon was noticed among CWHI by Kelly and Mousley (2001). Since NHC also experienced significant difficulty in this task, it could be attributed to developmental deficits (Epstein et al., 1994) which is a common factor in both the groups. This clearly signifies the importance of analytical and thinking strategies for solving arithmetic word problems (Hyde et al., 2003). An advantage in language and reading will not help to solve word problems (Kelly, Lang, & Pagliaro, 2003; Mousley & Kelly, 1998; Kelly & Mousley, 2001).

Analysis of errors committed and deficiencies exhibited while attempting the task

The common errors committed and deficiencies exhibited by children from both the groups while doing sums in ADT were analysed qualitatively. The analysis aimed at identifying and classifying the errors and deficiencies. The total number of children in each group was converted into percentages for the purpose of comparing. The results are given in Tables 8.8–8.12 for the two groups, with examples and probable reasons for each type of error/deficiency.

Table 8.8 Number concept

Sl No.	Error/deficiency	Example	% of children		Probable reason
			SS	RS	
1	Writing digit wise while writing in words	214 – Two one four	64	17	Poor understanding of place value and/or poor verbo-sequential processing
2	Delete, add or substitute digits while writing in numbers from words	Three hundred fourteen as 14 Sixty-nine as 699 Four hundred eighteen as 488	53	50	Poor understanding of place value and/or difficulty in coding the verbal information
3	Not attempted as the complexity of the task increases for most of the criterion measures	Did not attempt tasks where more than 2-digit numbers were given	32	28	Poor understanding of place value and/or avoiding as the complexity of the task increases

Table 8.9 Addition

Sl No.	Error/deficiency	Example	% of children		Probable reason
			SS	RS	
1	Forgets to add the carried digit to higher place	$\begin{array}{r} 85 \\ + 69 \\ \hline 144 \end{array}$	30	11	Poor working memory
2	Did not attempt when the numbers exceeded 2 digits and more than two numbers had to be added	$\begin{array}{r} 429 \\ 781 \\ 365 \\ \hline \end{array}$	28	50	Not learnt the procedure or avoiding the difficult task
3	Added all the numbers given in the statement problem though the numerical information is irrelevant to solve the problem	<p>Krishna had 12 Rupees. He bought 2 books for 5 Rs, 2 pencils for 3 Rs. How much money did he spend?</p> $12+2+5+2+3$	87	56	Difficulty in processing verbal numerical information

Table 8.10 Subtraction

Sl No.	Error/deficiency	Example	% of children		Probable reason
			SS	RS	
1	Forgets the number borrowed	$\begin{array}{r} 73 \\ -54 \\ \hline 29 \end{array}$	36	6	Poor working memory
2	Writes 0 as answer when 0 has to be subtracted from a digit	$\begin{array}{r} 84 \\ -60 \\ \hline 20 \end{array}$	53	39	Lack of '0' concept
3	Writes '0' when the corresponding digit in the minuend is greater than the subtrahend	$\begin{array}{r} 81 \\ -76 \\ \hline 10 \end{array}$	6	6	Not learnt operation of borrowing from the higher place
4	Subtracts the smaller number from the larger when the minuend is smaller than its corresponding digit in the subtrahend	$\begin{array}{r} 92 \\ -87 \\ \hline 15 \end{array}$	6	17	Not learnt operation of borrowing from the higher place
5	Added the numbers instead of subtracting in the statement problem	Savithri had Rs 25, she spent Rs 21. How much money does she have left? $25 + 21 = 46$	57	39	Not able to process verbal numerical information

Table 8.11 Multiplication

Sl No.	Error/deficiency	Example	% of children		Probable reason
			SS 1	SS 2	
1	Wrote the number as answer when multiplied with 0	$6 \times 0 = 6$	51	11	Lack of '0' concept
2	Adding the numbers instead of multiplying	$7 \times 2 = 9$	11	11	Does not know multiplication, or confusion of symbols + and \times
3	Could not perform multiplication involving more than one digit	Did not attempt sums like 21×7 , 70×8 , etc.	45	33	Not learnt the skill of multiplying 2-digit numbers
4	Adding all the numbers from the statement problem instead of multiplying	In a class there are 10 rows of chairs. Every row has 6 chairs. How many chairs are there altogether? $10 + 6 = 16$	28	33	Not able to apply suitable algorithm depending on the situation. Performing the easier task
5	Not able to solve statement problems	Did not attempt	60	39	Not able to process verbal numerical information

Table 8.12 Division

Sl No.	Error/deficiency	Example	% of children		Probable reason
			SS	RS	
1	Multiplying instead of dividing	$10 \div 5 = 50$	23	17	Does not know division, or does not know the meaning of the symbol
2	Could not perform division when more than one step is required	Did not attempt sums like $95/3, 7007/7$	81	83	Not learnt the procedure of division when more than one step is involved
3	Adding all the numbers from the statement problem instead of dividing	There are 59 students in a class. How many groups can be made of 7 boys and how many will be remaining? $59 + 7 = 66$	60	50	Not able to apply suitable algorithm depending on the situation. Performing the easier task
4	Not able to solve statement problems	Did not attempt	70	83	Not able to process verbal numerical information

Major findings and discussion in the error analysis stage

Number concept

The errors in writing digits were observed more among CWHI. As the task requires verbo-sequential processing where CWHI are found to perform below average (Craig & Gordon, 1988), recalling the name of the number in a sequence representing the place of the digit with suitable case markers (in Kannada) between each place might be the reason. Hence, they may simplify it just by writing the digits in words, not processing the number as a whole. This strategy is also adapted by NHC. The rest of the errors are committed by both groups almost equally. This shows that CWHI deal with same conceptual obstacles in understanding numerical concepts as hearing children and they deal with such obstacles in similar ways (Nunes & Moreno, 1998).

Fundamental operations

Forgetting to add the carried number and deduct the borrowed number was committed by a greater number of CWHI compared to NHC. As CWHI are deficient in inner speech (Hitch et al., 1983) or using working memory (Epstein et al., 1994) they may be more likely to commit this error. This suggests that the procedure of carry-over and borrowing is known to them, as the first step in the procedure is attempted correctly. It is also noted while analysing the difficulty, that the percentage of CWHI attaining mastery in these two tasks is greater compared to the NHC. A greater number of CWHI have attempted these items and hence the percentage of children committing the error is higher.

There is not much difference in the percentage of children committing errors/ deficiencies in multiplication and division, indicating similar lines of thought process. Many children did not attempt the multiplication and division items.

Arithmetic reasoning

The error of adding all the digits in the statement problem and adding instead of subtracting and multiplying is due to tackling a problem mechanically without trying to understand it. It is also noticed that a considerable number of children from both the groups did not attempt statement problems, especially under multiplication and division. The negative, disengaged approach to word problem-solving tasks was noted by Kelly and Mousley (2001). The fact that NHC also made this error indicates that the reason is not just language- or reading-based, but a lack of analytical thinking (Hyde et al., 2003).

Conclusion

In the present study, though CWHI had the dual disadvantage of sensory and environmental deprivation, there was no significant difference in their performance when compared with NHC who were drawn from an environmentally deprived

population. As the delay was noticed in both the groups, it could be attributed to developmental deficits due to environmental factors rather than hearing impairment. It was observed by Ramaa and Gowramma (2002) that a considerable number of socially disadvantaged children experienced difficulty in performing grade-appropriate mathematical tasks. As noticed by Zarfaty et al. (2004) this could be due to fewer opportunities to learn the culturally transmitted aspects of mathematical knowledge. Schoenfeld (2002) emphasized the importance of environmental factors based on data on disproportionate numbers of poor African-American, Latino, and Native American students dropping out of mathematics and performing below standard on tests of mathematical competency. A comparative study on the arithmetic performance of CWHI from special schools and day schools/inclusive schools from different socio-economic strata would be necessary for a clearer understanding.

For most of the items appropriate for grades I and II, 100% mastery was not achieved by either group. As the students are in grades IV and V, this could be considered as a serious difficulty in learning arithmetic. There is a clear indication that the difficulty level increases as the complexity of the task increases for both the groups in all the areas of arithmetic assessed. This must be viewed seriously as mathematics is a linear subject and this lag could have serious implications for future performance. This can be attributed as Schoenfeld (2002) also noted, to less exposure to mathematical experiences consequent of the environmental deprivation experienced by the economically and educationally disadvantaged families in this sample.

Most of the errors could be due to a poor understanding of the concept of place value. Since it is an abstract concept, all children find it difficult to understand at the primary school stage. The strength areas, like spatial representation (Zarfaty et al., 2004), visualization (Nunes & Moreno, 2002; Yathiraj, Gowramma, Prithi, & Vijetha, 2013), visuo-spatial skill (Craig & Gordon, 1988) and providing concrete experience (Vijayalakshmi & Gowramma, 2010), should be used to develop concepts. Remedial instruction at the right time would decrease the lag in learning and arrest further damage.

Neither of the groups in the present study exhibited errors in number concepts such as the rotation of digits (6 as 9), reversal of numbers (12 as 21) and reproducing the same numbers in the blanks when it was asked to continue the sequence (6, 7, . . . , . . . , as 6, 7, 6, 7), which were noticed by Gowramma (2005) among children with dyscalculia of the same grade level. In addition she noticed errors like adding from the left side, adding all the numbers ($21 + 15$ as $2 + 1 + 1 + 5$) which are due to difficulty in spatial organization and failing to grasp the logic. Most of the items on subtraction and multiplication were not attempted by children with dyscalculia. None of them attempted any item under division. This suggests that the brain and cognitive process disruption stemming from neurobiological deficit leading to difficulty in arithmetic are distinct (Ashkenazi, Black, Abrams, Hoefft, & Menon, 2013). Dyscalculia stems from a core deficit in processing quantity (Butterworth, Varma, & Laurillard, 2011). The difficulties and errors committed by CWHI and those from an environmentally deprived population, as observed in the present study, do not stem from a specific deficit in number processing. It can

be compared to the domain-general hypothesis of mathematical disability which posits that working memory (Rotzer et al., 2009; Toll, Van der Ven, Kroesbergen, & Van Luit, 2011) and attention (Ashkenazi & Henik, 2010a, 2010b) cause the deficits in the use of developmentally appropriate arithmetic procedures (Ashkenazi et al., 2013). Ramaa and Gowramma (2004) compared the performance of children with visual impairment, children with dyscalculia and normally developing children in number concept and found that children with visual impairment in primary level had attained mastery in number concept just like their fully sighted age peers, but children with dyscalculia, even at grade IV, had basic difficulties. This further strengthens the fact that sensory impairment per se is not the cause of arithmetic difficulties. Since the core brain areas and functional circuits involved in numerical cognition are unaffected in CWHI and environmentally deprived NHC, unlike children with dyscalculia, age-appropriate mathematical experience will help them learn grade-level arithmetic concepts. Comparative studies on larger samples can direct towards precise behavioral diagnosis for dyscalculia.

Without intervention the difficulties may persist for a longer duration, leading to additional problems. Schoenfeld (2002) cautions on the long-term implications of such deficits as it denies an important pathway to economic and other enfranchisement. In this context, it may be noted that children with dyscalculia were able to show grade-appropriate performance in arithmetic after following systematic remediation based on appropriate principles (Gowramma, 2005). Hence, there is a need to design appropriate remedial strategies suiting the special needs of children. It can be construed from the findings of the present study that CWHI have some areas of strength in addition to having difficulties in learning arithmetic. It was found that the performance of CWHI improved significantly when they were taught through play-way methods in primary school (Vijayalakshmi & Gowramma, 2010). Findings of a study by Mousley and Kelly (1998) demonstrated that problem-solving performance of CWHI can be enhanced through exposure to a number of analytical strategies. Craig and Gordon (1988) suggest using the right hemisphere strengths of visuo-spatial skills to compensate the left hemisphere deficit in verbo-sequential processing. These studies offer a bridge between research and practice and the potential for making effective use of the strengths to surmount the deficiencies and provide the quality of education a student deserves. Matching the educational intervention to the needs of each student implies finding pedagogical methodologies that are effective across the spectrum of abilities. Such a change in the classroom is the need of the hour when the policy is to educate all the children under the same roof, irrespective of their abilities and socio-economic background.

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